



What is algebra?

Algebra is a **system for working with general quantities taking on different values**, known as variables. These quantities are often denoted by single letters, e.g. x .

Advice: A lot of the algebra skills are based on arithmetic (i.e. numeracy skills). If you struggle with arithmetic, have a look at our study notes on Numeracy Skills as you may need to brush up on this first.

Topics in algebra include:

- Simplifying expressions
- Solving equations involving one variable
- Factorising expressions
- Problems involving powers of variables
- Problems involving algebraic fractions

Why should I learn algebra?

If you need to study algebra as part of your degree course, it is probably because algebra is an important skill for your course profession.

Apart from this, some people enjoy learning to do algebra because it provides a language for describing regular structures and processes, such as patterns.



Activity

Advice: Find some examples of using algebra in your course or ask your instructor to give you some.

What's involved in learning algebra?

Firstly, the most important skill in algebra is to become fluent **in thinking of a variable as a general quantity** which can take on any value.

Secondly, as in arithmetic, expressions involving arithmetic operations should be thought of as both a **process** and **the result of a process** at the same time.



Example

Example: Just as $4 + 5$ can be thought of as:

- Count from 1 to 4 then count from 1 to 5 (a **process**)
- A known fact, i.e. 9 (the **result** of a process)

In the same way, $3 + 2x$ can be thought of as both:

- The **process** of adding together 3 and 2 times a general quantity called x
- The **result** of adding together 3 and 2 times x



Example

Another key skill in algebra is **rewriting algebraic expressions based on rules**.

Example: $4y + 5y$ can be rewritten as $9y$.

Common errors in learning algebra

According to research, students generate three main types of error when learning to do algebra:

1. Using arithmetic rules in the wrong way

This kind of error is to do with taking a technique from arithmetic and using it in algebra in the wrong way.

Example: whilst $\frac{3x}{x}$ can be simplified to 3, $\frac{3x+2y}{x+y}$ **cannot** be simplified to $3+2$ because this

algebraic fraction is **not** equivalent to $\frac{3x}{x} + \frac{2y}{y}$, so simplification does not work in the same

way. In general, you can add together and separate the tops (numerators) of fractions but not the bottoms (denominators).



Example



Advice

Advice: This example can be understood by replacing x and y with numbers and comparing with rules in arithmetic. This is a good general technique if you are struggling to understand algebraic rules.



Activity

Activity: Show that $\frac{3x+2y}{x+y}$ is not equal to $\frac{3x}{x} + \frac{2y}{y}$ when $x=4$ and $y=5$.

2. Concept changes between arithmetic and algebra

Not every concept in arithmetic is the same in algebra. For example, **place value** is used in arithmetic to represent whole numbers larger than 9 but putting symbols together in algebra means they should be **multiplied together**.



Example

Example: Consider the problem:

$$\text{Solve for } x: \quad 4x = 46$$

The 4 on the left hand side **cannot** be cancelled with the 4 on the right hand side (to give $x=6$) because $4x$ means 4 times x whereas 46 means forty six.

3. Procedural errors

These are to do with the **incorrect execution of rules**, rather than misunderstanding what they mean.



Example

Example: for the problem:

$$\text{Solve for } x: \quad \frac{5}{x+2} + \frac{5}{x-2} = 3$$

A possible **error** would be to rewrite the equation as:

$$5(x-2) + 5(x+2) = 3$$

This shows a correct application of a rule to multiply out both fractions on the left hand side of the equation but an **incorrect** application of the rule on the right hand side as it should be multiplied by $(x+2)(x-2)$. This error might occur if you have only encountered problems where the fractions are on opposite sides of the equation.

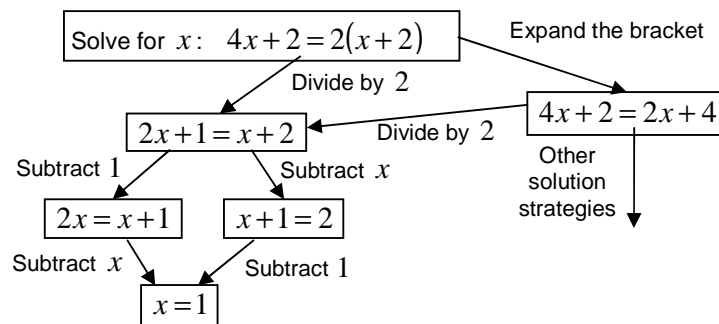
Problem solving in algebra

There are often many different ways to solve algebraic problems. Part of the skill in doing algebra is to choose the simplest method. This only comes with practice.



Example

Example: This diagram shows some of the possible ways of simplifying the problem given in the top box.



Reference

Suggested further reading

Mason, J., Graham, A & Johnston-Wilder, S. (2005) *Developing Thinking in Algebra*, London: Paul Chapman Publishing.

Academic Skills Centre Library and Learning Resources
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